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REPORT OF INVESTIGATIONS/1989

A Simplex-Method-Based Algorithm for Determining the Source Location of Microseismic Events

By Jennifer S. Riefenberg

BUREAU OF MINES

UNITED STATES DEPARTMENT OF THE INTERIOR

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CONTENTS

Page

ILLUSTRATIONS

TABLE

UNIT OF MEASURE ABBREVIATIONS USED IN THIS REPORT

ft	foot	pct	percent
ft/s	foot per second	s	second

A SIMPLEX-METHOD-BASED ALGORITHM FOR DETERMINING THE SOURCE LOCATION OF MICROSEISMIC EVENTS

By Jennifer S. Riefenberg[1]

ABSTRACT

The U.S. Bureau of Mines conducts basic and applied research related to predicting, eliminating, and/or controlling rock bursts in underground hard-rock mines and coal bumps in underground coal mines. An important element in this research is the development of a reliable and efficient microseismic source location technique. This report presents the results of a Bureau study to develop and evaluate a new algorithm to locate microseismic events based on the Simplex method, a powerful linear programming technique. The technique was tested on both field data and simulated data. Microseismic event source locations obtained with this new Simplex-based algorithm were shown to have less error than those locations obtained using a least squares method. It was concluded that the Simplex approach to source location of microseismic events can be used effectively in microseismic monitoring.

[1]Geophysical engineer, Denver Research Center, U.S. Bureau of Mines, Denver, CO.

INTRODUCTION

Rock bursts, the dynamic and usually destructive failures of rock mass under load, are becoming more commonplace as underground mines endeavor to extract ore from increasing depths. Rock burst problems occur most often where high-stress conditions exist. Depth, geologic factors, and mining activities contribute to the load on a rock mass. Each year, some underground mine fatalities and a number of serious injuries, as well as extensive damage to mining equipment and facilities, can be directly attributed to unexpected rock burst activity. For example, in 1987, 1 fatality and 30 injuries in underground hard-rock and coal mines in the United States were directly attributable to rock burst or coal bump activity, according to the U.S. Mine Safety and Health Administration. Not only are rock failures costly in terms of human life and health, but these failures can be economically disastrous. A single rock burst or coal outburst may completely destroy an active mining area, thus rendering future reserves in that area unobtainable. To control or minimize damage from rock bursts, the U.S. Bureau of Mines is currently conducting research in microseismicity to determine the suitability of its applications to rock burst prediction. This work is in support of the Bureau's mission to provide the mining industry with a safer working environment.

Microseismic monitoring, in simple terms, is the process of listening to a rock mass to determine where microscale failure is occurring. The emanation of elastic waves generated by microfracturing of the rock mass is often associated with changing load. Locating the source of microseismic events resulting from microfracturing of the rock mass is the purpose of microseismic monitoring. If an anomalous increase in the number of microseismic events occurs in a localized zone over a relatively short time period, concentrations of stress may be occurring. Decisions might be made to take action to alleviate the potentially hazardous stress conditions.

To monitor microseismic events, a geophone or accelerometer array is installed at known locations in the rock mass surrounding an area to be monitored. Microseismic event waveforms detected at these geophones are amplified, validated, and stored on a computer. A source location for each event is then determined through the relative arrival times of signals recorded on each channel and the corresponding geophone locations. Relative arrival times are the times at which a first break in the signal waveform occurs; in addition, arrival times are recorded relative to the first geophone activated by a single microseismic event.

The commonly used least squares location method suffers from an exaggerated error effect; in the presence of arrival times containing large error, its use can result in the solution's being weighted in favor of the most erroneous arrival time. The approach presented in this report does not indiscriminately weight the data and, therefore, should provide more accurate results than the least squares method. The Simplex method, a readily available, fast, and widely accepted algorithm for solving linear programs, is a logical solution to the problem of microseismic event source location.

BACKGROUND

The Bureau has been involved in microseismic research for about 15 years. In that time a least squares algorithm has become the most accepted and practical method for locating microseismic event sources. A few other source location algorithms have been used to some extent, but these algorithms, based on trial and error, are very inefficient. At present, the algorithm used in all field applications is an iterative least squares method developed by Leighton and Duvall[2] in 1972. This report presents the results of a comparison between this iterative least squares method and a new algorithm, presented in this report. The essence of the new approach is to transform the least squares problem into a linear program that can be solved via the Simplex method.

Present research in microseismicity is fast moving toward a much higher level of sophistication. Not only are fully digital microseismic waveform capture and analysis now possible, but with the additional implementation of triaxial geophones, a whole new level of information may be gained. In addition, with the type of data collected in a digital, truly three-dimensional manner, many analysis techniques used in the field of seismology may be employed. With improvements in data quality, as well as an increase of information, source location solutions may be improved, and additionally, focal mechanisms that generate the microseismic event may be delineated. The field of seismology incorporates a number of source location algorithms, usually based on a nonlinear velocity model. In the future it is hoped that the Bureau may employ some of these available algorithms, relaxing the assumption of a constant velocity, and further improve the solution to microseismic event source location.

[2]Leighton, F., and W. I. Duvall. A Least Squares Method for Improving the Source Location of Rock Noise. BuMines RI 7626, 1972. 19 pp.

TECHNICAL APPROACH

The first step in developing a new algorithm was to inspect the present least squares equation and to determine if it could be altered in some way that would retain all of the original information. Since an ordinary least squares approach suffers from an exaggerated residual error effect, the natural question was whether one can lessen or remove this exaggerated error in the alteration of the least squares equation. While there are a number of alternatives toward removing the exaggeration in error, it was decided to try an approach that would lend itself to a linear programming solution.

The least squares equation is simply the sum of squares of the residuals. In the Bureau's application, these residuals are the differences in distance between the actual data and a model solution. The approach taken to lessen the exaggerated error was to sum the magnitudes of the residuals rather than sum the squares of the residuals. It can be readily seen that there is no loss of information in using this approach. On a historical note, this approach of summing magnitudes is credited to Pierre-Simon Laplace and actually predates the least squares approach with which Sir Isaac Newton is credited.

The Simplex-method-based approach consists of two distinct stages. The first stage, included in appendix A, is a computer program that accepts as input an arrival time data set and manipulates this information into the form of a linear program. This linear program is then input in the second stage, a linear programming package that utilizes the Simplex method algorithm, and the resultant output from this package, the solution to microseismic event source location. Although the linear programming package used in this report was written by Bureau personnel, it is not included as an appendix, because commercial linear programming packages would more than suffice and are readily available on a variety of computer systems. Appendix B presents an example set of field data and the resulting linear program.

With the completed algorithms programmed and tested, the comparative analysis was undertaken. A number of simulated data sets were tested, as were a number of actual arrival time data sets that had been collected in a deep silver mine. The source location solutions obtained through these tests were then compared with results obtained using the Bureau's iterative least squares algorithm, and differences in the resultant errors in solution were quantified and contrasted.

DISCUSSION

With exact data and ideal assumptions (for example, assuming a constant velocity through a homogeneous medium, direct travel paths of the microseismic wavefront, etc.), a source location can be guaranteed. Given that field data are not exact nor are the assumptions made perfectly consistent with physical reality, there is inherent error associated with any computed solution. Obtaining the highest possible quality of signals captured and the highest possible accuracy of arrival times from these signals is necessary to ensure source locations with a minimum of error. It is also of utmost importance that a source location algorithm be as accurate as possible. Although all location algorithms are dependent upon the quality of field data, the Simplex method approach, which does not indiscriminately weight erroneous data, appears to give locations superior to those given by the least squares method.

An initial approach begins with two equations, which form the basis of the algorithm for source location using the Simplex method approach. These equations are

$$d = \sqrt{(a_i - x)^2 + (b_i - y)^2 + (c_i - z)^2}, \tag{1}$$

and

$$d = vt, \tag{2}$$

where d = distance,

a_i, b_i, c_i = ith spatial coordinates,

x, y, z = reference spatial coordinates,

v = velocity,

and t = time.

Let a_o, b_o, c_o be the coordinates of the geophone nearest the source, let a_i, b_i, c_i be the ith geophone coordinates, and let x, y, z be the coordinates of the unknown microseismic event source. Then using equation 1 to express the distance from the source to the closest geophone d_o, and from the source to the ith geophone d_i, the following are obtained (see also figure 1):

$$d_o = \sqrt{(x - a_o)^2 + (y - b_o)^2 + (z - c_o)^2},$$

$$d_i = \sqrt{(x - a_i)^2 + (y - b_i)^2 + (z - c_i)^2}. \tag{3}$$

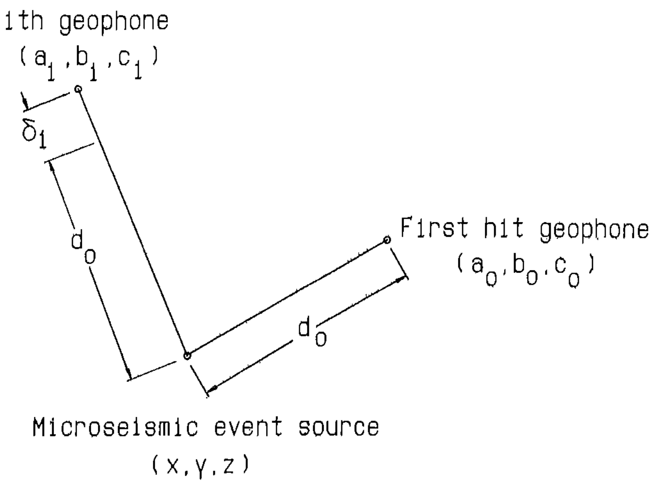

Figure 1.–Graphic representation of distances between microseismic event source and two distinct geophones.

Additionally, let v be the seismic wave velocity (assumed constant along a direct travel path). Let t_o be the travel time from the microseismic event source to the nearest geophone, and let t_i be the difference between the travel time from the source to the nearest geophone and the travel time from the source to the ith geophone; then,

$$\hat{d}_i = v(t_o + t_i), \tag{4}$$

where $\hat{d}_i$ = distance from microseismic event source to ith geophone.

From figure 1 and equations 3 and 4,

$$\sqrt{(x - a_o)^2 + (y - b_o)^2 + (z - c_o)^2} + \delta_i = \sqrt{(x - a_i)^2 + (y - b_i)^2 + (z - c_i)^2}. \tag{5}$$

where δ_i = vt_i.

Squaring both sides of equation 5 and rearranging results in the following:

$$\delta_i^2 + a_o^2 + b_o^2 + c_o^2 - a_i^2 - b_i^2 - c_i^2 - 2[(a_o - a_i)x + (b_o - b_i)y + (c_o - c_i)z] = -2\delta_i\sqrt{(x - a_o)^2 + (y - b_o)^2 + (z - c_o)^2}. \tag{6}$$

To simplify, let

$$D_i = a_o^2 + b_o^2 + c_o^2 - a_i^2 - b_i^2 - c_i^2,$$

$$\Delta a_i = a_o - a_i,$$

$$\Delta b_i = b_o - b_i,$$

and

$$\Delta c_i = c_o - c_i.$$

Dividing equation 6 by δ_i, and employing these simplifications,

$$\delta_i + (D_i/\delta_i) - (2/\delta_i)[\Delta a_i x + \Delta b_i y + \Delta c_i z] = -2\sqrt{(x - a_o)^2 + (y - b_o)^2 + (z - c_o)^2}. \tag{7}$$

Because the distance from the source to the 0th geophone is unknown, it is necessary to work instead with differences in arrival times between the 1st and the jth geophones. If t_o is the arrival time to the closest geophone activated by the microseismic event and t_k is the arrival time to the kth geophone triggered, then $\delta_k = \Delta t_k v$, where $\Delta t_k = t_k - t_o$. Multiplying through by v, the resulting equation is as follows:

$$v^2(\Delta t_1 - \Delta t_j) + (D_1/\Delta t_1) - (D_j/\Delta t_j) = (2/\Delta t_1)[\Delta a_1 x + \Delta b_1 y + \Delta c_1 z] - (2/\Delta t_j)[\Delta a_j x + \Delta b_j y + \Delta c_j z] \tag{8}$$

for j = 2,...,m (m = n - 1, where n is the number of geophones triggered), or

$$W_j + T_j v^2 = A_j x + B_j y + C_j z, \tag{9}$$

where
$W_j = D_1/\Delta t_1 - D_j/\Delta t_j$,

$A_j = 2[\Delta a_1/\Delta t_1 - \Delta a_j/\Delta t_j]$,

$B_j = 2[\Delta b_1/\Delta t_1 - \Delta b_j/\Delta t_j]$,

$C_j = 2[\Delta c_1/\Delta t_1 - \Delta c_j/\Delta t_j]$,

and $T_j = \Delta t_1 - \Delta t_j = t_1 - t_j$.

Normalizing equation 9, for ease in computation, gives

$$\alpha_j x + \beta_j y + \gamma_j z = p_j, \tag{10}$$

where
$\alpha_j = A_j/\sqrt{A_j^2 + B_j^2 + C_j^2}$,

$\beta_j = B_j/\sqrt{A_j^2 + B_j^2 + C_j^2}$,

$\gamma_j = C_j/\sqrt{A_j^2 + B_j^2 + C_j^2}$,

and $p_j = (W_j + T_j v^2)/\sqrt{A_j^2 + B_j^2 + C_j^2}$.

Theoretically, equation 10 is an equality, but in the presence of actual data it rarely is. To account for this apparent discrepancy, a "residual" is defined as the difference between the actual data and a computed solution from this model equation (fig. 2).

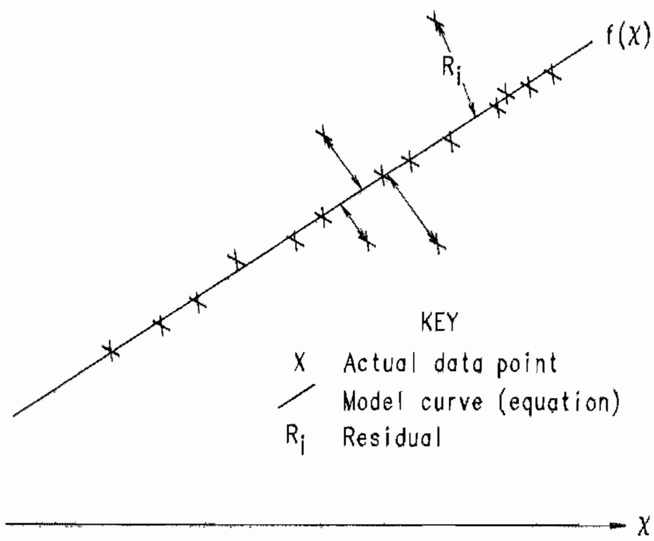

Figure 2.–Two-dimensional representation of a residual R_i, where the residual is the (perpendicular) distance between the ith actual data point and the model equation. Note that in this depiction there are four data points that appear rather erroneous.

From figure 2, it should be quite clear that summing the residuals themselves is inappropriate. Residuals on one side of the model curve will be of the opposite sign from that of residuals on the other side of the model equation. Because of this sign dependency, one must work instead with magnitudes or squares of these residuals. By minimizing the sum of the magnitudes (or squares) of the residuals, one is obtaining a "best fit" curve to the data. The model equation is the resulting equation representing this best fit curve.

In mathematical terms, the residual is as follows:

$$R_j = \alpha_j x + \beta_j y + \gamma_j z - p_j,$$

where R_j = the residual.

A typical least squares approach is based on minimizing the sum of squares of the residuals. A least squares problem is then

$$\min \sum_{j=2}^{m} R_j^2 \tag{11}$$

To use the Simplex method, rather than minimizing the sum of squares of the residuals, the approach is to minimize the sum of the absolute values (magnitudes) of the residuals; i.e.,

$$\min \sum_{j=2}^{m} |R_j| \tag{12}$$

or

$$\min_{\underline{x}} \sum_{j=2}^{m} |\alpha_j x + \beta_j y + \gamma_j z - p_j| \tag{13}$$

Now, let $a_j\underline{x} = \alpha_j x + \beta_j y + \gamma_j z$,

where $\underline{x}$ is the column vector (x, y, z),

and let $q_j = a_j\underline{x} - p_j$.

Substituting the above into equation 13 gives

$$\min \sum_{j=2}^{m} |q_j|. \tag{14}$$

Convert the minimization equation 14 to a linear program by making the change of variables as follows:

$$\text{let } |q_j| = q_j^+ + q_j^-. \tag{15}$$

where $q_j^+ = (|q_j| + q_j) / 2$,

and $q_j^- = (|q_j| - q_j) / 2$.

The problem may now be written

$$\min \sum_{j=2}^{m} (q_j^+ + q_j^-), \tag{16}$$

subject to

$$a_j\underline{x} - q_j^+ + q_j^- = p_j, \tag{17}$$

where $\underline{x}$ is free,

$q_j^+ >= 0$

and $q_j^- >= 0$ for every j.

The constraining equation 17, written in matrix notation, becomes

$$A\underline{x} - \underline{q}^+ + \underline{q}^- = \underline{p}.$$

Finally, the linear program generated, in matrix notation, is

$$\min \{\underline{q}^+ + \underline{q}^-\}, \tag{18}$$

subject to

$$[\,A \mid -I \mid I\,]\,[\underline{x}\ \underline{q}^+\ \underline{q}^-]' = \underline{p}, \tag{19}$$

$\underline{q}^+ >= 0$,

$\underline{q}^- >= 0$,

and $\underline{x}$ is free,

where A = matrix composed of components a_{ij},

I = identity matrix,

-I = negative identity matrix,

[ ]' = transpose vector-matrix.

Now that the problem is posed as a linear programming problem, it can be solved using the Simplex method as described by Bazaraa and Jarvis.[3] The Simplex method is well known for its efficiency and is readily available on most computer systems as it is the basis on which most linear programming software packages perform.

Both methods of solution, the least squares method and the Simplex-method-based algorithm, are concerned with solving an overdetermined system of equations, i.e., more equations than unknowns. It is extremely rare to find an exact solution to an overdetermined system of equations; thus, there is error associated with a computed solution.

The Simplex method minimizes an objective function subject to a system of constraining equations, i.e., a linear program. In this case the objective function is the linear equation $\underline{q}^+ + \underline{q}^-$. The error value associated with the Simplex method is a function of how much slack must be taken into account to solve the system of equations at an optimum value while minimizing an objective function. The system of constraining equations determines a convex, feasible region over which the objective function can be minimized. Since minimization is not performed over a sum of squares, the Simplex method is affected less by erroneous data then is the least squares method and, thus, should render better solutions. The error value from the Simplex method approach is the same as the error defined by the minimized sum of the magnitudes of the residuals since these are equivalent processes.

An erroneous data point is a data sample that appears disassociated from the majority of data samples. For example, with a data plot in which there appears to be a linear trend, if one data point lies far from the trend, it can usually be assumed to be erroneous. The process of determining whether an element of data is erroneous is relative and can be very subjective. In microseismic event data, an erroneous data point is an arrival time difference that is disproportionate with the other arrival time differences in the set in that it may be, for example, an order of magnitude 10 times greater than the other arrival time differences.

Often in microseismic monitoring events occur outside of the geophone array. Both the Simplex-method-based and least squares location algorithms are fully capable of locating such events, referred to here as "outliers." Problems arise because of the decrease in the quality of data as one moves away from the array. In addition, velocity assumptions and microseismic wave travel path assumptions can become grossly inaccurate, which has a negative effect on any source location algorithm based on these assumptions. Again, the Simplex-method-based algorithm should produce more accurate results than the least squares method when locating events outside the array, simply because it is less affected by erroneous data (whether due to inaccurate assumptions or poor signal quality) then is the least squares method.

RESULTS

An algorithm developed by the Bureau and written in the C programming language was used to test the Simplex-method-based approach. An arrival time data set, which contained both field data and simulated data (and a constant velocity), was tested using both the Simplex and least squares algorithms. Table 1 contains the results from this comparison. Where simulated data were used, actual source locations from which the arrival time data were generated are also tabulated. Error values (or residual error) associated with the computed solutions in the two algorithms are presented in this table as well. Table 1 also contains results from the same arrival time data set, but with erroneous arrival times removed. Figure 3 is a graphic representation of table 1.

In the majority of cases, the Simplex solutions on field data have associated with them lower error values. This lower error is consistent with expected results. As one can readily see, with exact arrival time (simulated) data, both the Simplex and least squared algorithms perform ideally. Even when a 15-pct random error is introduced into the arrival time data, both algorithms perform well.

Differences between the least squares algorithm and the Simplex algorithm solutions appear to be due to the weighted error in the least squares method. Discrepancies apparently due to weighted error are readily seen when one compares the errors for unprocessed and preprocessed field data in table 1 (or figure 3 left and right), where, in the latter, the data were preprocessed to remove erroneous data. When the erroneous arrival times are removed, the two algorithms generally give locations that are more consistent with each other. The Simplex solutions, as would be expected, do not change nearly as drastically as the least squares solutions.

[3] Bazaraa, M. S., and J. J. Jarvis. Linear Programming and Network Flows. Wiley, 1977. 565 pp.

Table 1.–Comparison of microseismic event source locations calculated with Simplex-method-based algorithm and with the least squares algorithm

Case No.	Number of arrival times	Difference between solutions, ft	Simplex solution: Coordinates x, y, z	Simplex solution: Error, ft	Least squares solution: Coordinates x, y, z	Least squares solution: Error, ft
			FIELD DATA			
1	5	0	1543, 387, 4213	0	1543, 387, 4213	122
2	8	81	1648, 482, 4378	61	1726, 464, 4391	113
3	8	54	1654, 502, 4393	56	1689, 472, 4421	144
4	6	58	1704, 455, 4324	20	1713, 457, 4381	287
5	7	29	1657, 477, 4279	24	1665, 500, 4294	26
6	6	134	1748, 435, 4593	13	1748, 441, 4459	289
7	9	1947	1703, 556, 4364	4	2916,1015, 5816	376
8	8	1115	1701, 559, 4394	2	2390, 815, 5233	224
9	8	20	1613, 478, 4307	46	1618, 491, 4322	13
10	10	12	1616, 478, 4342	1	1615, 466, 4340	2
11	8	192	1551, 487, 4096	105	1640, 428, 4255	25
			SIMULATED DATA			
12	12	0	1575, 350, 4300	0	1575, 350, 4300	0
13	12	0	1600, 500, 4300	0	1600, 500, 4300	0
14	12	0	1675, 400, 4300	0	1675, 400, 4300	0
15[1,2] ...	12	9	1565, 355, 4289	7	1573, 351, 4287	12
16[1,3] ...	12	6	1601, 499, 4302	11	1600, 500, 4300	6
17[1,4] ...	12	3	1669, 399, 4306	16	1670, 399, 4303	19
			PREPROCESSED FIELD DATA[5]			
1	5	0	1543, 387, 4213	0	1543, 387, 4213	122
2	6	1	1650, 434, 4348	0	1651, 434, 4348	0
3	6	94	1668, 463, 4365	30	1689, 466, 4457	33
4	5	0	1704, 455, 4324	0	1704, 455, 4324	22
5	6	37	1700, 504, 4361	14	1721, 497, 4391	31
6	5	134	1748, 435, 4593	13	1748, 441, 4459	289
7	7	973	1754, 561, 4598	3	1754, 562, 5566	821
8	7	53	1766, 570, 4166	3	1752, 567, 4217	98
9	6	4	1616, 478, 4308	8	1611, 480, 4307	16
10	9	6	1610, 470, 4335	1	1613, 469, 4340	3
11	6	27	1675, 458, 4332	6	1665, 459, 4308	13

[1]15-pct random error in arrival time data.
[2]Coordinates from which data generated: 1575, 350, 4300.
[3]Coordinates from which data generated: 1600, 500, 4300.
[4]Coordinates from which data generated: 1675, 400, 4300.
[5]Arrival time data preprocessed to eliminate outliers.

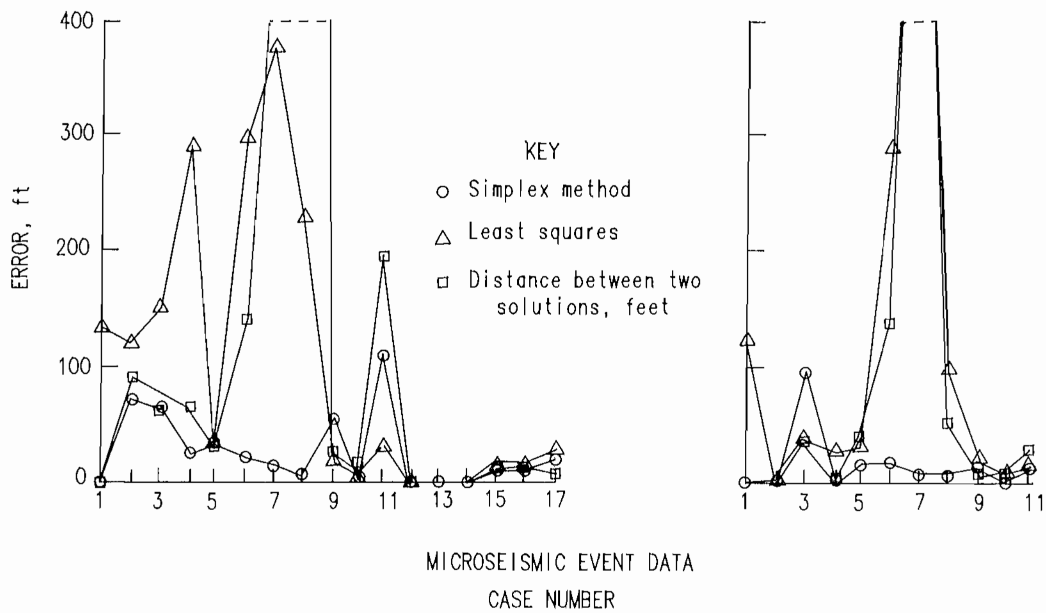

Figure 3.–Error in microseismic event source locations. Simplex method versus least squares method. Left, field and simulated data. Right, preprocessed field data. (See table 1).

CONCLUSIONS

Microseismic event location solutions provided by the Simplex-method-based algorithm have significantly lower error than those provided by the iterative least squares method. In the presence of raw field data, nearly all of the Simplex solutions (more than 80 pct) had substantially lower error in the source location than did the corresponding least squares solutions. After the data were preprocessed, virtually all of the Simplex solutions had lower errors than did the least squares solutions. Results based on the lower errors and the less drastic changes in solution using the Simplex-method-based approach lend credence to it as a superior method of solution and a method of possibly greater accuracy. While a superior location algorithm aids in improving the solution to a microseismic event source location, one cannot disregard the importance of collecting data of the highest quality to ensure accurate locations. It is concluded that the Simplex-method-based approach is a viable algorithm for microseismic event source location.

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APPENDIX A.–PROGRAM LISTING ON FIRST-STAGE PROCESS

The following program represents the first-stage process (fig. A-1) in the algorithm developed in this study to calculate a microseismic event source location. This program requires the following as input: the number of geophones triggered in a single event, the velocity through the medium at which the signal travels, the x-y-z coordinates of each geophone activated by the event, and the arrival time (in seconds) corresponding to each geophone activated. Information from at least five geophones is needed to produce a solvable linear program. Note that with only five geophones activated, an exact solution will be produced as the system of equations is not overdetermined (i.e., three equations and three unknowns).

It is recommended that more than five geophone arrival times be used because the increase of information would have a rather large effect on the solution. For example, as shown in figure A-2, when there are only two points, the obvious best fit curve is a straight line between the two points, and this solution has zero residual error. When a third point is added, the options for a fitted curve have increased. With this added information, the best fit solution, though now having a nonzero residual error, may be much more realistic.

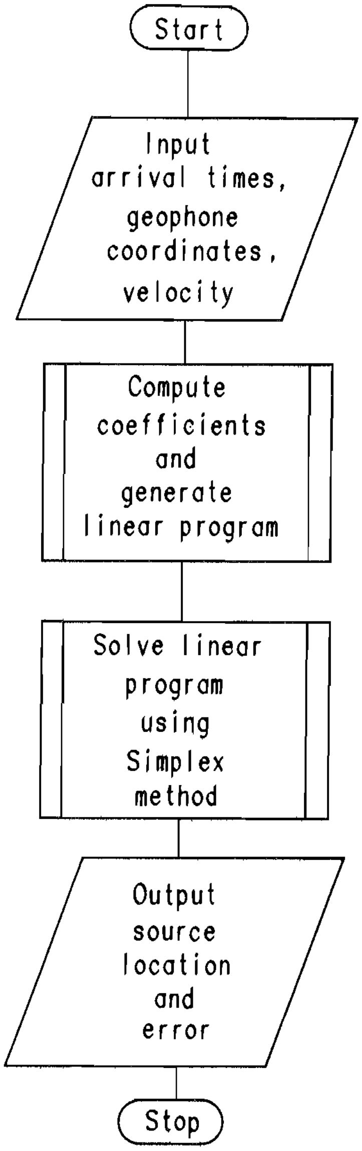

Figure A-1.–General flow diagram depicting two-stage process in determining a microseismic event source location via the Simplex-method-based algorithm.

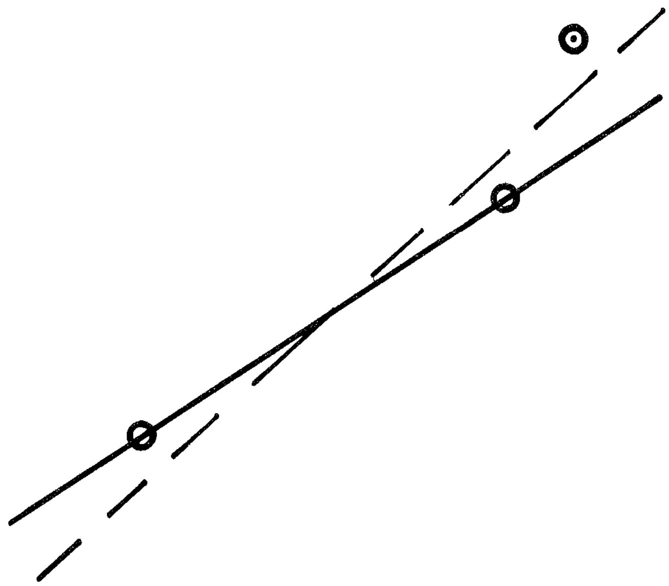

Figure A-2.–Two best fit curves: (solid) through two points (exact fit) and (dashed) through three points. Shown is the dependence of the best fit curve on the number of data samples.

```
#include <stdio.>
#include <math.h>

/* The following program calculates the needed Tableau for a system of equations to be solved using the Simplex
   algorithm.

   The required input information is the number of geophones triggered, the velocity through the medium, the x-y-z
   coordinates of each geophone and the arrival time corresponding to each geophone.

        Programmer: J. Riefenberg
        Date: 7/19/88

*/

double velocity,AoSqr,BoSqr,CoSqr,delta_a[20],delta_b[20],delta_c[20];
double [20],A[20],B[20],C[20],T[20],W[20],alpha[20],beta[20],agamma[20];
double p[20],delta_time[20];
double ax[20],by[20],cz[20],Tableau[50][50],time[20],radical[20];
int i,j,k,No_of_phones,m,n,Np;

Calculate_coefficients()
{
    AoSqr       = ax[0] * ax[0];
    BoSqr       = by[0] * by[0];
    CoSqr       = cz[0] * cz[0];
    for (i=1;i<=No_of_phones;i++)   {
        delta_a[i] = ax[0] - ax[i];
        delta_b[i] = by[0] - by[i];
        delta_c[i] = cz[0] - cz[i];
        D[i] = (AoSqr + BoSqr + CoSqr) - ((ax[i]*ax[i]) + (by[i]*by[i]) + (cz[i]*cz[i]));
    }
    for  (j = 2;j<=No_of_phones:j++)   {
        A[j] = 2.0 * ((delta_a[i] / time[1]) - (delta_a[j] / time[j]));
        B[j] = 2.0 * ((delta_b[i] / time[1]) - (delta b[j] / time[j]));
        C[j] = 2.0 * ((delta_c[i] / time[1]) - (delta_c[j] / time[j]));
        delta_time[j] = time[1] - time[j];
        W[j] = ((D[1] / time[1]) - (D[j] / time[j]));
        radical[j] = sqrt((A[j]*A[j]) + (B[j]*B[j]) + (C[j]*C[j]));
        p[j-2] = (W[j] + delta_time[j] * (velocity*velocity)) / radical[j];
        alpha[j-2] = A[j] / radical[j];
        beta[j-2] = B[j] / radical[j];
        agamma[j-2] = C[j] / radical[j];
    }
}   /* end of Calculate_coefficients function   */

main()
{
printf(" Input number of geophones hit:   ");
    scanf("%d",&Np);
    No_of_phones = Np - 1;
    printf("\n Input velocity through medium (ft/s):    ");
    scanf("%1f",&velocity);
    for (k=0;k<=No_of_phones;k++)   {
        printf("\n for geophone with arrival time %2d",k);
      printf("\n     enter x,y and z coordinates (x y z): ");
        scanf("%1f %1f %1f",&ax[k],&by[k],&cz[k]);
        printf("\n enter corresponding arrival time (sec):   ");
        scanf("%1f",&time[k]);
        printf("\n");
```

```
    }
    Calculate_coefficients();
    n = (No_of_phones - 1) * 2 + 3;
    m = No_of_phones - 1;
    for (i=0;i<=m;i++)
        for (j=0;j<=n;j++)
                Tableau[i][j] = 0.0;
    for (i=0;i<=m-1;i++)    {
                Tableau[i][0] = alpha[i];
                Tableau[i][1] = beta[i];
                Tableau[i][2] = agamma[i];
                Tableau[i][n] = p[i];
    }
    for (i=0;i<=m-1;i++)    {
        for (j=3;j<=((n-3)/2 + 2);j++)
                if (i+3 == j)
                    Tableau[i][j] = -1.0;
        for (j=((n-3)/2 + 3);j<=n-1;j++)
                if ((i+<n-3)/2 + 3) == j)
                    Tableau[i][j] = 1.0;
    }
    for (j = 3;j<=n-1;j++)
        Tableau[m][j] = 1.0;
    for (i=0;i<=m;i++)   {
        for (j=0;j<=n;j++)
        printf(" %5.2f ".Tableau[i][j]);
        printf("\n");
    }
}
/*  End of main  */
```

APPENDIX B.–EXAMPLE USING FIELD DATA

The following is an example of field data and the resulting linear program generated by the program in appendix A.

Sample input (velocity is 18,500 ft/s):

Arrival time, s	Geophone	Geophone coordinates
0.0	6	1570, 464, 4330
0.000132	4	1621, 460, 4295
0.001764	5	1602, 530, 4295
0.002044	9	1680, 470, 4295
0.003640	1	1516, 463, 4295
0.004148	2	1681, 553, 4295
0.004960	3	1707, 461, 4238
0.006064	12	1620, 610, 4295
0.008036	8	1567, 291, 4295
0.019040	10	1514, 589, 3995

Sample output (linear program in tableau form):

-0.82	0.15	0.55	-1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1122.74
-0.80	0.08	0.60	0	-1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1335.16
-0.84	0.06	0.54	0	0	-1	0	0	0	0	0	0	0	1	0	0	0	0	0	998.58
-0.81	0.12	0.58	0	0	0	-1	0	0	0	0	0	0	0	1	0	0	0	0	1255.14
-0.82	0.07	0.57	0	0	0	0	-1	0	0	0	0	0	0	0	1	0	0	0	1158.99
-0.82	0.12	0.56	0	0	0	0	0	-1	0	0	0	0	0	0	0	1	0	0	1172.07
-0.83	0.02	0.56	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	1	0	1097.82
-0.84	0.08	0.53	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	1	1001.41
0.00	0.00	0.00	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.00

The tableau is a common form with which linear programs are depicted when they are to be solved using the Simplex method. A commercial software package will probably require some modification of this form as its required input. This tableau contains the information necessary to implement a commercial package. The first eight rows in the tableau are the constraints induced by the arrival time data set. The rightmost column is the right-hand side of constraint matrix (the constraining values). The bottom row is the objective function to be minimized, and the lower right corner element of the tableau is the current (initially infeasible) value of the objective function. The naturally occurring constraints that (1) x, y, and z are all free (or unconstrained), and (2) q_{j+} and q_{j-} are positive, need also be specified for a commercial package.

The resulting solution to the linear program will contain values of all constrained variables, most importantly the x, y, and z coordinates and the minimized objective function value (the error in solution).